ECON 306
Mathematical Worksheet 1

Key mathematical concepts

1) Continuous and discrete variables

A continuous variable is one for which, within the limits the variable ranges, any value is possible. For example, the variable "Time to solve an anagram problem" is continuous since it could take 2 minutes, 2.13 minutes etc. to finish a problem. The variable "Number of correct answers on a 100 point multiple-choice test" is not a continuous variable since it is not possible to get 54.12 problems correct. A variable that is not continuous is called discrete.

2) Dependent variable and independent variable:

2.a) Definition. When you have an equation such as $y = f(x)$ usually you call x the independent variable or the input variable and y the dependent variable or the output variable. A good way to figure out which one is which is that, when you are analyzing a problem like the apartments problem in the textbook, the question you want to answer will be a question about your dependent variable (the number of apartments demanded) and the information in the question will be about the independent variable (the price of the apartments).

2.b) Graphs.
When drawing the graph of an equation in mathematics people usually draw the dependent variable (y) on the vertical axis and the independent variable (x) on the horizontal axis.
In Economics (by convention), when we draw the graph of a demand or supply curve, we usually the dependent variable on the horizontal axis. Consider the following demand function:

(Figure 1)

$$Q = 100 - 20P \qquad OR \qquad P = 5 - Q/20$$

In the graph we can see that the dependent variable is (Q)uantity, which is graphed in the horizontal axis while (P)rice is on the Vertical Axis. This is purely by convention. The important thing to remember is which variable is dependent and which is independent.
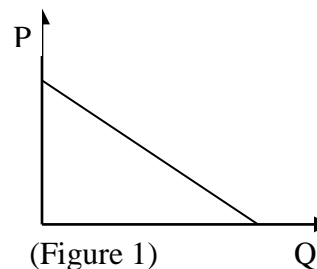
3) A function is a relation that uniquely associates members of one set with members of another set. A key thing to remember about a function is that for any number for the independent variable you get exactly one value for the dependent variable (that is, they relate a value of X to a unique value of Y). One example can be obtained from the following equation: $Y = X^2$
In this equation, for each value of X, we get exactly one value of Y, but if we start with a positive value of Y and then solve for X we get:

$$X = \sqrt{Y}$$

In this case X becomes the dependent variable and for each value of Y we get two values of X (the positive and the negative values of the root).

A graph that is a function

Y
X

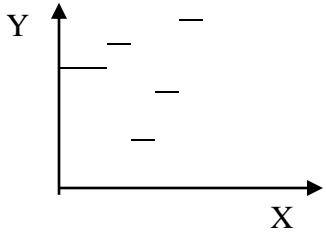**A graph that is <u>not</u> a function**

Y
X
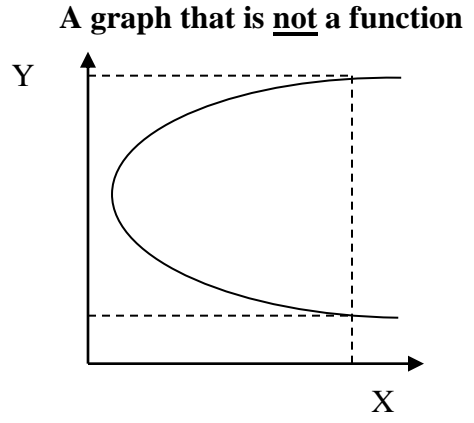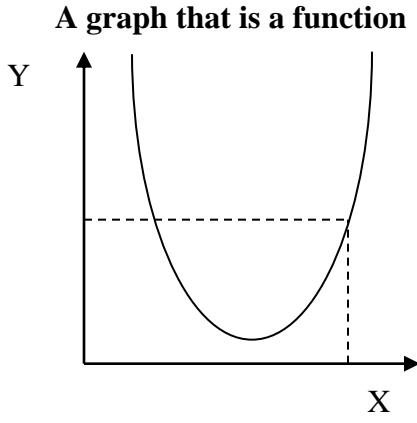
4) Step function: a function which is defined throughout some interval I and is constant on each one of the finite set of nonintersecting intervals whose union is I.
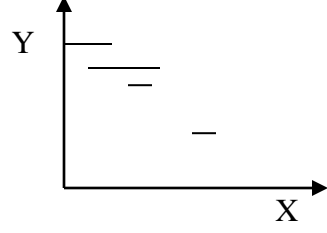Here is an example of a step function.

Y
X
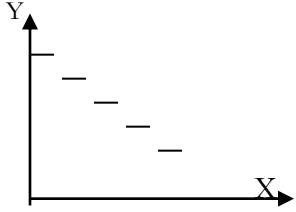
In this graph if y was the dependent variable it is a step function because for each value of x we get only one value of Y. If x was the dependent variable it would not be a function anymore as for some values of y we get an infinite number of values for x.

A graph that is a step function

Y
X

A graph that is <u>not</u> a step function

Y
X

5) Equations and identities

Equation (=): is only valid for some values of the variables.
Identity (≡): is valid for all values of the variables.

The following table presents some examples of identities and equations

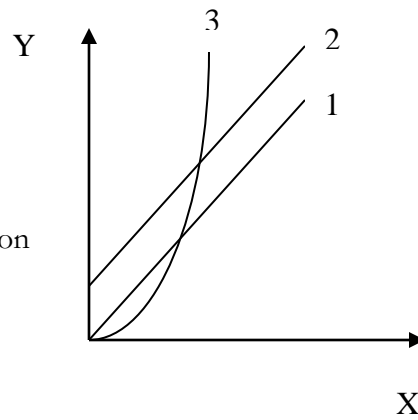| Identity	Equation
$Ln(A/B) \equiv Ln(A)-Ln(B)$	$8 = 5X$
$2X \equiv 4X/2$	$3 = Ln (Y)$
$(A-B)(A+B) \equiv A^2-B^2$	$5 = Sin(X)$
$Tan(A) \equiv Sin(A)/Cos(A)$	$(Y+6)*7 = 2$
$X \equiv X$	$Y = 2X$

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6) Linear and nonlinear functions

If function's graph is a line, it's called a "linear function".
1) $Y=2X$
2) $Y=2X+3$
3) $Y=X^2$

Equation 1 is a linear function, equation 2 is a linear function and equation 3 is a non-linear function.

7) Changes and rates of change

ΔX denotes a change in the variable X. ΔY denotes a change in the variable Y. The rate of change of the variable Y with respect to X is:

$$\frac{\Delta Y}{\Delta X} = \frac{f(X + \Delta X) - f(X)}{\Delta X}$$

In the case of a line (linear function), the rate of change of the dependent variable with respect to a change in the independent variable is constant.

In the case of a non-linear function, the rate of change of the dependent variable depends on the initial value of X and the value of ΔX. Below are examples (using the functions presented in 6)).
1) $\Delta Y/\Delta X = 2$
2) $\Delta Y/\Delta X = 2$
3) $\Delta Y/\Delta X = 2X + \Delta X$

As you can see, the rates of change of functions 1 and 2 are constant, and the rate of change of function 3 can vary (if we change the initial value of X and/or the value of ΔX).

8) Slopes and intercepts

The "slope of a curve between two points" is the rate of change of the dependent variable with respect to the independent variable. This is easy to picture in the case of a line (since it is constant), but for a curve, the slope can change as the independent variable changes.
Consider equation number 3 above
3) $Y=X^2$
In this case, we can find the slope of this curve between two particular points on the curve (as we did in 7). It is the same as the slope of a line which passes through those two points.

Some examples of a slope of a curve between two points